AP Calculus BC

Improper Integrals

1) $\int_2^\infty \frac{5}{x^2}\,dx$

$\lim_{b\to\infty} \int_2^b 5x^{-2}\,dx$

$\lim_{b\to\infty} \left[-5x^{-1}\right]_2^b$

$\lim_{b\to\infty} \left[-\frac{5}{b} + \frac{5}{2}\right]$

$\boxed{\frac{5}{2}}$

2) $\int_{-\infty}^{-27} x^{-1/3}\,dx$

$\lim_{a\to-\infty} \int_a^{-27} x^{-1/3}\,dx$

$\lim_{a\to-\infty} \left[\frac{3}{2}x^{2/3}\right]_a^{-27}$

$\lim_{a\to-\infty} \left[\frac{3}{2}(-27)^{2/3} - \frac{3}{2}a^{2/3}\right] = -\infty$

DIVERGES

3) $\int_{-\infty}^{-1} -2x^{-2}\,dx$

$\lim_{a\to-\infty} \int_a^{-1} -2x^{-2}\,dx$

$\lim_{a\to-\infty} \left[2x^{-1}\right]_a^{-1}$

$\lim_{a\to-\infty} \left[-2 - \frac{2}{a}\right] = \boxed{-2}$

4) $\int_5^\infty \frac{4}{x^2+5x+6}\,dx = \int_5^\infty \frac{4}{(x+3)(x+2)}\,dx$

$\frac{A}{x+3} + \frac{B}{x+2} = \frac{4}{(x+3)(x+2)}$

$A(x+2) + B(x+3) = 4$

$\underline{x=-2}$ $\quad$ $\underline{x=-3}$

$B=4$ $\quad$ $-A=4 \to A=-4$

$\lim_{b\to\infty} \int_5^b \left[\frac{-4}{x+3} + \frac{4}{x+2}\right] dx$

$\lim_{b\to\infty} \left[-4\ln(x+3) + 4\ln(x+2)\right]_5^b$

$\lim_{b\to\infty} \left[-4\ln(b+3) + 4\ln(b+2) - (-4\ln 8 + 4\ln 7)\right]$

$\lim_{b\to\infty} \left[4\ln\frac{b+2}{b+3} + -4\ln\frac{7}{8}\right]$

$0 \quad + \quad 4\ln 7/8 \quad = \boxed{4\ln 7/8}$

5) $\int_{15}^\infty x\ln(15x)\,dx$

$\lim_{b\to\infty} \int_{15}^b x\ln(15x)\,dx$

$u = \ln(15x) \qquad v = \frac{1}{2}x^2$

$du = \frac{15}{15x}\,dx \qquad dv = x\,dx$

$\lim_{b\to\infty} \left[\frac{1}{2}x^2\ln(15x)\Big|_{15}^b - \int_{15}^b \frac{1}{2}x\,dx\right]$

$\lim_{b\to\infty} \left[\frac{1}{2}x^2\ln(15x) - \frac{1}{4}x^2\right]_{15}^b$

$\lim_{b\to\infty} \left[\frac{1}{2}b^2\ln(15b) - \frac{1}{4}b^2 - \left(\frac{1}{2}(15)^2\ln(15)^2 - \frac{1}{4}(15)^2\right)\right]$

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6) $\int_0^7 (2x+3)(x^2+3x)^{-1/2}\,dx$

$\lim_{a\to 0^+} \int_a^7 (2x+3)(x^2+3x)^{-1/2}\,dx$

$\lim_{a\to 0^+} \left[2(x^2+3x)^{1/2}\right]_a^7$

check
$\frac{1}{2}(x^2+3x)^{1/2}(2x+3)$

$\lim_{a\to 0^+} \left[2\sqrt{70} - 2(a^2+3a)^{1/2}\right]$

$\boxed{\sqrt{70}}$

7) $\int_0^{16} \frac{1}{\sqrt{16-x}}\,dx$

$\lim_{b\to 16^-} \int_0^b (16-x)^{-1/2}\,dx$

$\lim_{b\to 16^-} \left[-2(16-x)^{1/2}\right]_0^b$

$\lim_{b\to 16^-} \left[-2(16-b)^{1/2} + 2(16)^{1/2}\right] = \boxed{8}$

8) $\int_{-\infty}^1 \theta e^\theta\,d\theta$

$\lim_{a\to -\infty} \int_a^1 \theta e^\theta\,d\theta$

D	I
$+\ \theta$	e^θ
$-\ 1$	e^θ
$+\ 0$	e^θ

$\lim_{a\to -\infty} \left[\theta e^\theta - e^\theta\right]_a^1$

$\lim_{a\to -\infty} \left[e - e - (ae^a - e^a)\right] = \boxed{0}$